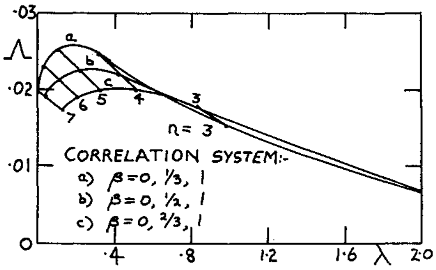

Fig. 4 Variation of Λ with $\lambda (m = 3)$.

Figures 3 and 4 show the corresponding results for $m = 2$ and $m = 3$, respectively. As in Fig. 2, the various correlation systems show wide differences up to the time of Λ_{max} but they agree well subsequently. The values of n corresponding to Λ_{max} are seen to be, approximately, 2, 3, and 4 for the cases $m = 1$, 2, and 3, whereas in Table 1 the corresponding values for K_{max} are $n = 1$, 1.1, and 1.2, respectively.

To summarize, the correlation system has been shown to have limited usefulness. The parameter K in Eq. (5) is not a reliable guide to the magnitude of the reduction in torsional stiffness since maximum values of K and Λ do not correspond, and the latter is a more useful parameter to assess the reduction, Eq. (11).

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Method of Belotserkovskii for Asymmetric Blunt-Body Flows

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Introduction

There are several available methods of solution for symmetric supersonic blunt-body flows. Many of these methods are described by Hayes and Probstein.[1] On the other hand, for asymmetric flows there are very few available methods, even when the asymmetry does not introduce an additional independent variable as in the case of plane flow. One of the problems has been the determination of the stagnation streamline and, consequently, the value of the entropy on the body.[2] A popular hypothesis has been to assume that the streamline that wets the body in an asymmetric flow is the one that crosses the shock at right angles and, hence, has the maximum entropy in the shock layer. In particular, this hypothesis was made in applying the Belotserkovskii one-strip method to the problem of flow over an axisymmetric body at angle of attack. However, it is clear that the stagnation streamline ought to be determined by the method of solution and should not require an a priori hypothesis. Swigart[4, 5] has developed an inverse method‡ for symmetric bodies at small angles of attack which has this feature. In this method, no hypothesis is made regarding the position of the maximum-entropy streamline; its position in the flow is determined as part of the solution and it is in fact not the stagnation streamline.

The subject of this paper is the application of the Belotserkovskii one-strip method to plane asymmetric flows. This is a direct method in the sense that the shock and flow field are determined for a given body shape. For the one-strip method, it will be shown that the stagnation streamline is a straight line, and hence there are sufficient relations at the stagnation streamline so that an a priori hypothesis concerning the body entropy does not have to be made. This result also applies to the method of Refs. 2 and 3 for axisymmetric bodies at angle of attack so that in those works it would have been possible to determine the body entropy in the forementioned way.

Analysis

The method of Belotserkovskii is the application of Dorodnitsyn's method of integral relations to blunt-body flows and is described in Ref. 1 on pages 214–226. References to the papers of Dorodnitsyn and Belotserkovskii are given in Ref. 1. Briefly, the method consists of putting the governing fluid dynamic equations in "divergence form," assuming suitable linear approximations in the direction normal to the body surface, and then integrating in the normal direction from the body to the shock. There then results a set of ordinary differential equations in the direction of the body surface for certain dependent variables. Such a formulation has been made by Xerikos and Anderson[6] for symmetric bodies of general shape. We shall adopt their notation and equations in this paper.

The coordinate system and notation are shown in Fig. 1. The quantity s is the distance along the body surface, n is the distance normal to the surface, and $R(s)$ is the radius of curvature of the body at s. v_n is the component of flow in the n direction, and v_s is the component perpendicular to that direction. $\theta(s)$ is the body angle, $\chi(s)$ the shock angle, and $\delta(s)$ the shock-layer thickness. The subscript zero refers to quantities on the body and the subscript δ to quantities on the shock. The stagnation point on the body surface is the point at which $v_{s_0} = 0$; the subscript st refers to quantities at the stagnation point.

The linear approximations in the n direction employed by Xerikos and Anderson are

$$\begin{aligned} t &= t_0 + (n/\delta)(t_\delta - t_0) \\ z &= (n/\delta)z_\delta \\ G &= G_0 + (n/\delta)(G_\delta - G_0) \end{aligned} \qquad (1)$$

where

$$\begin{aligned} t &\equiv (1 - V^2)^{1/(\gamma - 1)} \qquad V^2 = v_s^2 + v_n^2 \\ z &\equiv \rho v_s v_n \\ G &\equiv (1/R)\{\rho v_s^2 + [(\gamma - 1)/2\gamma]p\} \end{aligned} \qquad (2)$$

Upon substitution of Eq. (1) in the governing fluid dynamic equations and integration in the n direction from the body to the shock, Xerikos and Anderson obtained the following simultaneous ordinary differential equations:

$$\frac{d\delta}{ds} = \frac{1 + (\delta/R)}{\tan(\theta + \chi)} \qquad (3)$$

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‡ In the inverse method the shock-wave shape, freestream conditions, and angle of incidence of the shock wave are specified, and the corresponding body and flow field are to be determined.

$$\frac{d\chi}{ds} = \frac{F_1 + F_2(d\delta/ds)}{F_3} \qquad (4)$$

$$\frac{dv_{s_0}}{ds} = \frac{F_4 + F_5(d\delta/ds) + F_6(d\chi/ds)}{F_7} \qquad (5)$$

for the dependent variables δ, χ, and v_{s_0}. The F's are complicated algebraic expressions given by Xerikos and Anderson which contain implicitly the body entropy in addition to the dependent variables and s.

For the symmetric case, it is known that the stagnation point occurs at $s_{st} = 0$, that $\chi_{st} = 0$, that the normal at the stagnation point is in fact the stagnation streamline, and, therefore, that the body entropy is that behind the normal shock. The problem is then a two-point boundary-value problem: the shock standoff distance δ_{st} must be determined such that the flow is regular at the sonic point. This latter condition requires some explanation. The denominator F_7 in Eq. (5) is

$$F_7 = \frac{\delta}{2}\,(1 - v_{s0}^2)^{(2-\gamma)/(\gamma-1)}\left(1 - \frac{\gamma+1}{\gamma-1}\,v_{s_0}^{\,2}\right) \qquad (6)$$

At the sonic point, $s = s^*$, the second factor is zero, and, therefore, in order that dv_{s_0}/ds remain finite, the numerator must be zero at the sonic point. This necessary condition for regularity at the sonic point supplies a relationship that completes the boundary conditions for the problem.

On the other hand, for the asymmetric problem, since there are no symmetry conditions at the stagnation point $v_{s_0} = 0$, there is much less a priori information about the stagnation point and streamline. In addition to the detachment distance δ_{st}, the location of the stagnation-point s_{st} and the shock angle χ_{st} are unknown. Furthermore, the shape of the stagnation streamline is unknown, and, therefore, the angle at which it crosses the shock is unknown, and hence the body entropy is unknown. Thus, there are three additional unknowns at the stagnation point: s_{st}, χ_{st}, and the body entropy. Of course, for the asymmetric problem there are two sonic-line regularity conditions, so that the system seems to have two indeterminate boundary conditions. However, it follows from the linear approximation assumed in the second of Eq. (1) that the stagnation streamline is a straight line. This can be easily seen if that equation is rewritten with the appropriate substitution from Eq. (2):

$$\rho v_s v_n = (n/\delta)\rho_\delta v_{s_\delta} v_{n_\delta} \qquad (7)$$

It is assumed that there is some point on the shock where the streamline direction immediately behind the shock is perpendicular to the body surface. It seems clear that this is so for bodies without slope discontinuities in the vicinity of the nose, i.e., for nonpointed bodies. At such a point on the shock, $v_{s_\delta} = 0$, and therefore $\rho v_s v_n$ vanishes all along that normal from the shock to the body. It is intuitively evident and can be shown that ρv_n does not vanish along the normal. It therefore follows that v_s must vanish, and the normal is the stagnation streamline. Thus it is shown that in the one-strip approximation the stagnation streamline is a straight line. This fact alone supplies two additional boundary conditions at the stagnation point: for any stagnation-point s_{st}, the flow direction behind the shock is known, and therefore, from the shock relations, the shock angle χ_{st} (assumed to be the strong shock solution) and the body entropy are known. Thus the shock angle χ_{st} and the body entropy are functions of the location of the stagnation-point s_{st}, and the unknown boundary values are reduced to s_{st} and δ_{st}.

Discussion

The conclusion that the stagnation streamline is a straight line is not so much a physical result as it is a consequence of the approximations involved in the application of the one-strip Belotserkovskii method. What has been shown is that the approximations of the one-strip method, which have been found suitable for symmetric flows, imply that the stagnation streamline is straight for asymmetric flows.

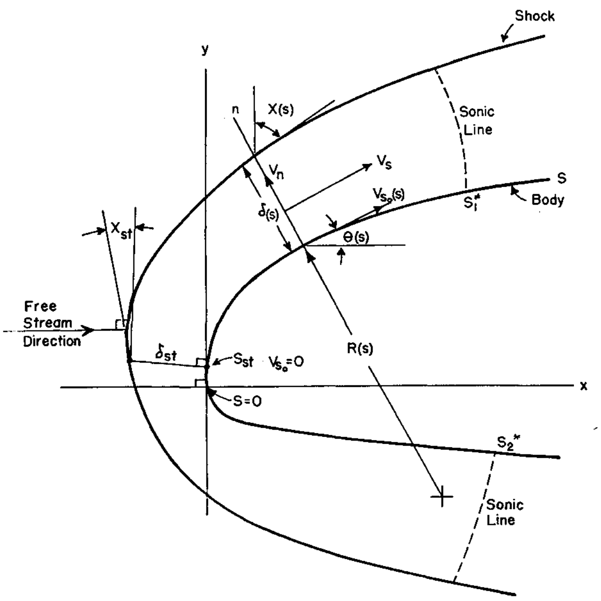

Fig. 1 **Asymmetric plane flow.**

If the nature of the approximations are altered, then it may not follow that the stagnation streamline is straight. Clearly, if the family of coordinate lines perpendicular to the body surface are curved rather than straight, the arguments of the preceding section still apply, so that the stagnation streamline will be one of the coordinate lines. That is, the stagnation streamline will be curved rather than straight. Thus, a physical result depends on the choice of coordinate system. To justify this result, it would be necessary to show that straight line coordinates yield a better approximation for the one-strip method than do curvilinear coordinates. On the other hand, curvilinear normal coordinates would require additional information that would lead to the selection of a specific curvilinear coordinate system.

Further, instead of a linear approximation for $z = \rho v_s v_n$, it might be assumed in some other form that we will represent by

$$z = f(n/\delta, z_\delta) \qquad (8)$$

It is, of course, necessary that f satisfy the following conditions:

$$f(0, z_\delta) = 0 \qquad z_\delta = f(1, z_\delta) \qquad (9)$$

If f also has the property that

$$f(n/\delta, 0) = 0 \qquad (10)$$

then the stagnation streamline will coincide with a normal coordinate line.

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